Gluons in the proton

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Outline

1. Introduction
2. Gluons in the vacuum
3. Unpolarized gluon distribution in the proton
4. Polarized gluon distribution and model calculations
5. Other information about the polarized gluons
6. Summary

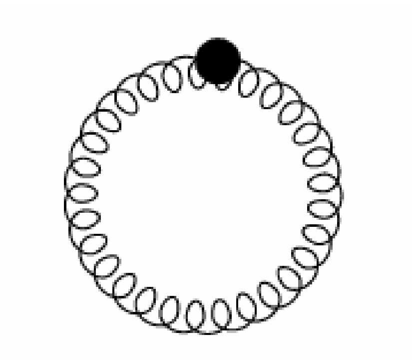
Introduction

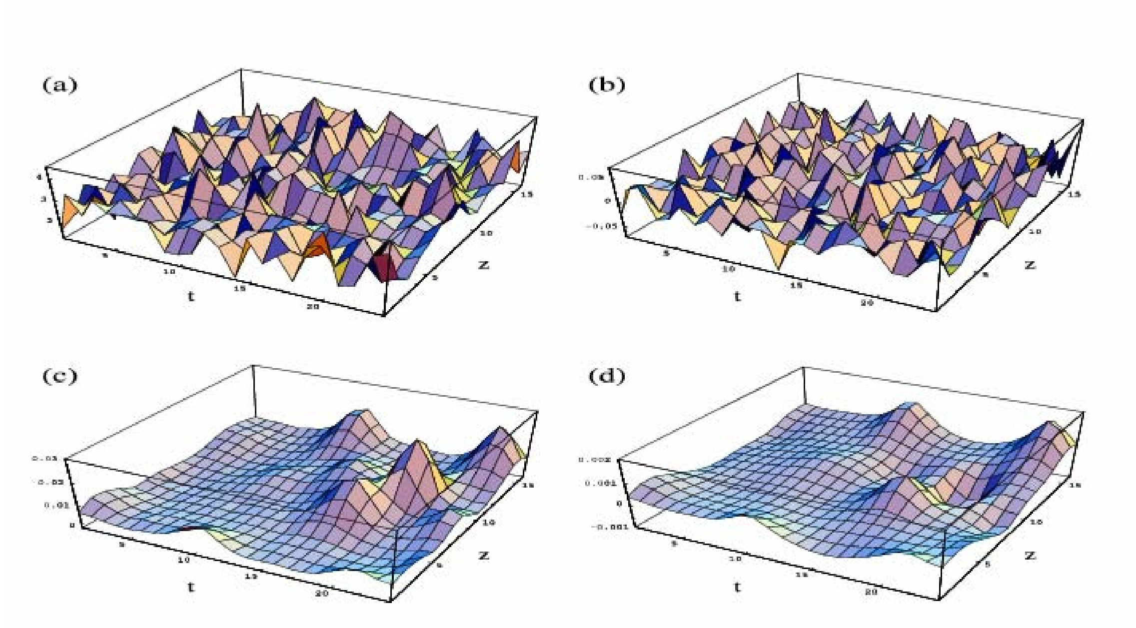
- The gluon is the integral part of QCD and, of course, the structure of the proton.
  - Without gluons, the quarks in the proton will fly apart.
  - Without gluons, the mass of the proton will be just the sum of quark masses.
  - Dominant mechanism for SM Higgs production
  - …
- However, it is actually hard to see the glue in the proton directly
  - Does not couple to electromagnetism
  - Gluon degrees of freedom missing in hadronic spectrum.
  - …

Where is the glue?

Gluons in the vacuum

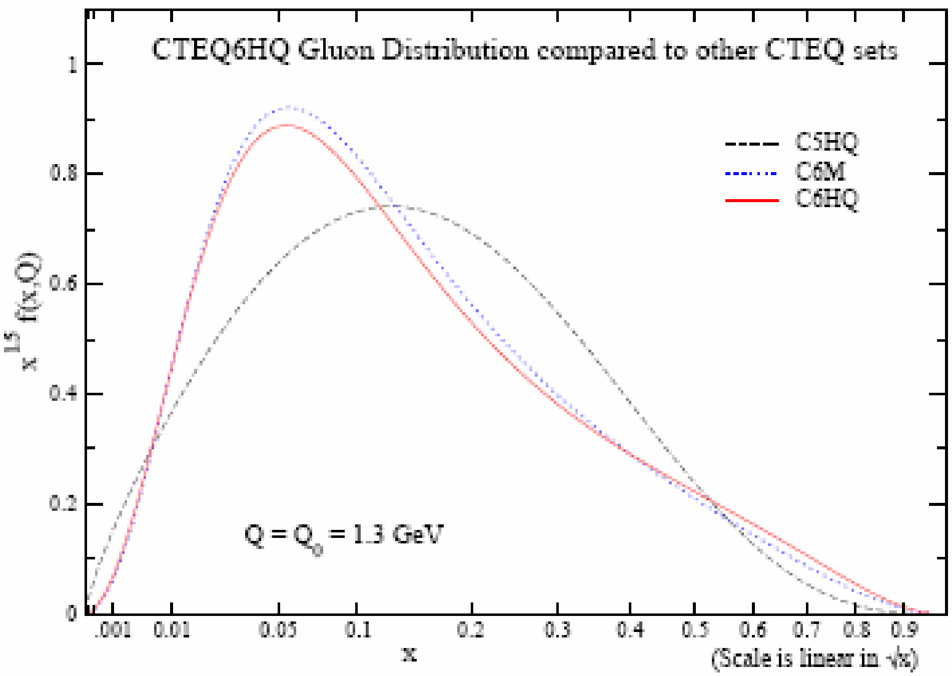
- In the large N_c limit, a scalar gluon operator of GG types goes like N_c^2 in the vacuum.
  - There are a lot of gluons in the vacuum which generates chiral symmetry breaking (instanton liquid) and color confinement (stochastic gluon vacuum)





Gluon Parton Distribution

- The most talked about is the gluon distribution in the proton and nuclei
- A lot of interesting things happen at small-x (parton saturation)

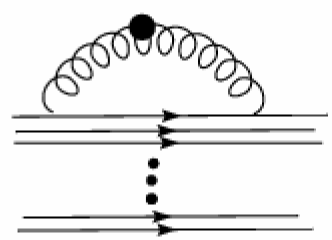


Large N_c limit

- However, in the proton the matrix element goes like N_c.
  - Introduction of valence quarks yields only a small change in the background gluon field.
  - The conclusion might be different at very small x «$1/N_c$
  - Gluon distribution in the nucleon goes like

$$g(x) = N_c^2 \, f(N_c x)$$

: fraction of nucleon momentum carried by gluon is a constant!



Nucleon Mass

- The hamiltonian of QCD comes from traceless and trace part of the energy momentum tensor

$$
\begin{aligned}
H'_q &= \int d^3\vec{x}\ \left[\bar{\psi}(-i\mathbf{D}\cdot\alpha)\psi + \frac{3}{4}\bar{\psi}m\psi\right], \\
H_g &= \int d^3\vec{x}\ \frac{1}{2}(\mathbf{E}^2+\mathbf{B}^2), \\
H'_m &= \int d^3\vec{x}\ \frac{1}{4}\bar{\psi}m\psi, \\
H_a &= \int d^3\vec{x}\ \frac{9\alpha_s}{16\pi}(\mathbf{E}^2-\mathbf{B}^2).
\end{aligned}
$$

  - The traceless part gluon energy contributes ($\sim N_c$) about 320 MeV/c^2 to the mass.
  - The trace part contributes (N_c) about 200 MeV/c^2 (dominated by gluon condensate in large N_c limit)

X. Ji, Phys.Rev.Lett.74:1071-1074,1995

Color Electric & Magnetic Fields

- One can solve the color electric and magnetic field in the nucleon from the above info

$$\langle P|\mathbf{E}^2|P\rangle = 1700 \text{ MeV},$$
$$\langle P|\mathbf{B}^2|P\rangle = -1050 \text{ MeV}.$$

  - The color electric field is stronger in the proton than that in the vacuum (strong coulomb field?)
  - The color magnetic field produced by the motion of valence quark is also strong, with the interesting feature that it almost cancels that field in the vacuum.

(A key to color confinement?)

Polarized gluon distribution $\Delta g(x)$

- The polarized gluon matrix elements go like N_c in the large limit. Thus the polarized part of the gluon field is
  - $1/N_c$ suppressed relative to the measurable gluon field in the nucleon
  - $1/N_c^2$ suppressed relative to the gluon fields in the vacuum.
  - $\Delta g(x) = N_c\, h(N_c x)$
- The total gluon helicity in the light-cone gauge $A^+ = 0$ is

$$\boxed{\Delta g = \langle PS | \vec{E} \times \vec{A} | PS \rangle}$$

Polarized Gluon Distribution $\Delta g(x)$ in Models

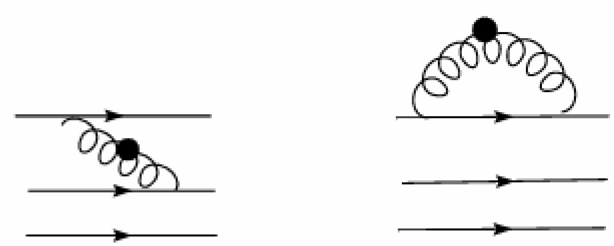
- In quark models, the gluons can be generated by quarks by solving color Maxwell's equation in linear approximation

$$D_\mu G^{\mu\nu} = j^\nu$$

- Of course, there are other gluons which are responsible for example, the potential between the quarks or bag confinement.
- However, one hopes that the correlation between the polarized gluon field and quark polarization can be generated correctly by models.
- Non-linear effects are ignored.

$\Delta g(x)$ in NR Quark Model

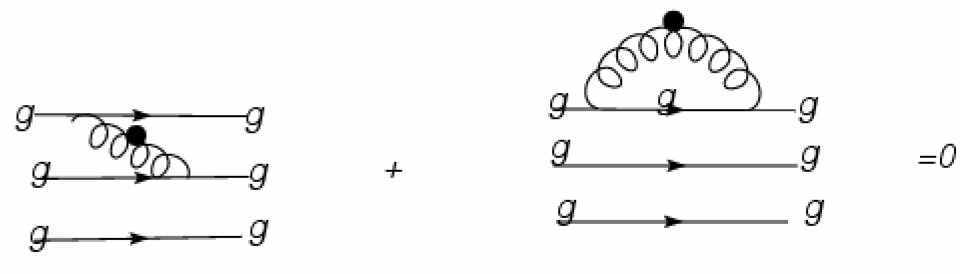
- In NR quark model, the spin-dependent color magnetic field is generated from the magnetic dipole of the quarks.
  - In MIT bag model, it is generated from the orbital motion of the nearly-massless quark.
- Two types of diagrams. The contribution from quark correlation and single quark.



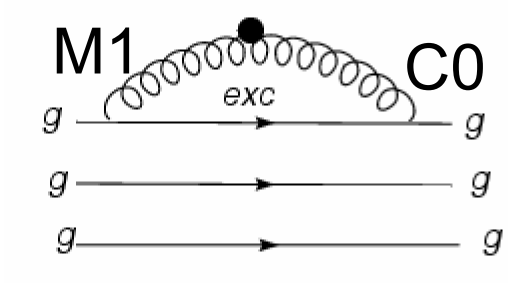
  - The first type is negative. (PRB365, R. L. Jaffe, 1996)

Δg(x) in NR quark model

- When the intermediate quark is taken to be at the ground state, the first type contribution cancel the second.



- Thus the most of the contribution comes from the contributions of the quarks in the excited states



Barone et al., PRB431,1998

  – The contribution is always positive: $\Delta g > 0$

A more general argument about the sign

- If the polarized gluons are generated from polarized quark in linear approximation, the vector potential circulates around the spin according to the right-handed rule.
- Because the coulomb gluon is always along the direction of r, EXA is always aligned with the spin of the quark.

Magnitude of Δg

- There are arguments from the discrepancy of $\Delta\Sigma$ from the quark model and EMC data and the QCD axial anomaly that Δg should be large: 1 to 2 units of hbar.
- The anomaly argument is at best controversial.
- Naturalness

$$\Delta\Sigma/2 + \Delta g + L_z = \tfrac{1}{2}$$

if Δg is very large, there must be a large negative L_z to cancel this---(fine tuning)

$$\Delta g < 0.5?$$

A QCD Sum Rule Calculation

- The total gluon contribution to the angular momentum of the proton is

  $$\int rX(EXB)d^3r$$

  A QCD sum rule calculation found that the contribution the contribution to the proton spin is about 0.25hbar

  I. Belitsky and X. Ji, PRL79, 1997

- Therefore if there is a cancellation, it must happen between Δg and orbital angular momentum of the gluon.

- Thus, there are theoretical arguments to favor a positive Δg with a magnitude <0.5 hbar.

Other probes of the polarized gluon fields

- Single-spin asymmetry
- Color-field polarizabilities
- Magnetic moment of the strange quark

Single Spin Asymmetry

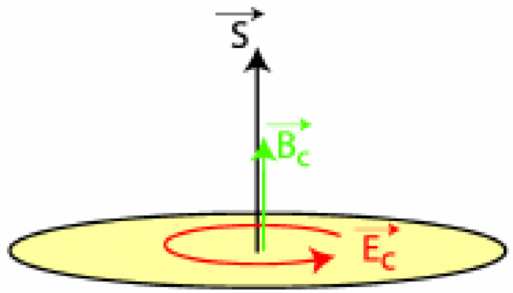
- In a polarized nucleon, the color electric field is polarized in a plane perpendicular to the direction of the polarization.
- This color electric field is the origin of the twist-three correlations

$$T_F(x_q, x_g) = \frac{1}{2\epsilon_\perp^{\beta\alpha} S_{\perp\beta}} \int \frac{d\zeta^- d\eta^-}{2\pi} e^{ik_q^+ \eta^-} e^{ik_g^+ \zeta^-} \langle PS|\overline{\psi}(0)\gamma^+ g F^{+\alpha}(\zeta^-)\psi(\eta^-)|PS\rangle \ .$$

- The direction of the field depends on the type of quarks which the gluon field is correlated--- M. Burkardt talk.

Second moment of g_2

Polarized Nucleon



Induced Color
Magnetic and Electric Fields

$$\mathbf{B}_C \sim \chi_B \mathbf{S}$$
$$\mathbf{E}_C \sim \chi_E \mathbf{S}$$

$$\boxed{d_2(Q^2) = \int_0^1 x^2 \left[2g_1(x,Q^2) + 3g_2(x,Q^2)\right] dx}$$

- At high Q^2, d_2 measures induced color field by target spin

$$d_2 = \frac{1}{8}(\chi_E + 2\chi_B)$$

Strange contribution to electromagnetic form factors

- Strange quarks have a vector current contribute both to the electromagnetic and weak neutral currents

$$J^{\mu}{}_{s} = \overline{s}\gamma^{\mu}s$$

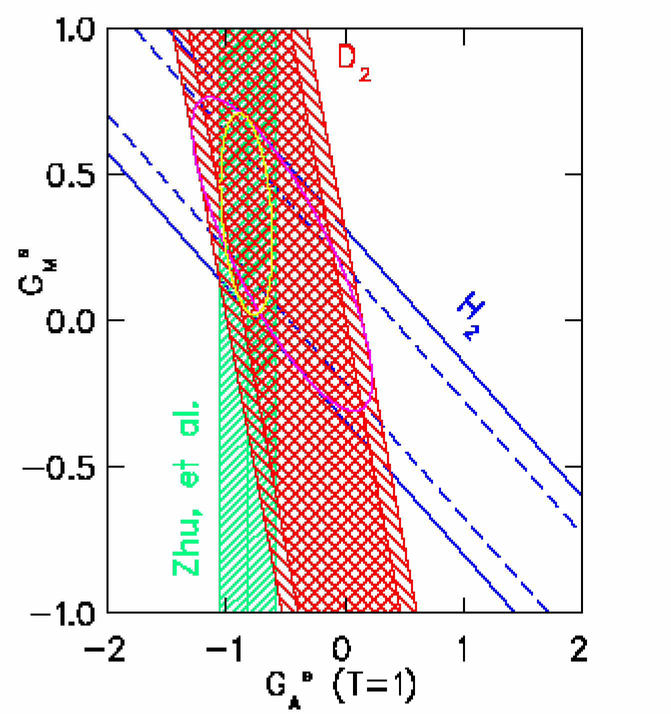
- Its contribution to the form factors

$$\begin{aligned} G^{\gamma}_{E,M} &= \frac{2}{3}G^{u}_{E,M} - \frac{1}{3}\left(G^{d}_{E,M} + G^{s}_{E,M}\right) \\ G^{Z}_{E,M} &= \left(1 - \frac{8}{3}\sin^2\theta_W\right)G^{u}_{E,M} + \left(-1 + \frac{4}{3}\sin^2\theta_W\right)\left(G^{d}_{E,M} + G^{s}_{E,M}\right) \end{aligned}$$

- G^s can be isolated from neutron & proton E&M form factors and weak current form factors, the latter can be extracted from parity-violating electron scattering.

SAMPLE

- Proposed around 1990 by R. McKeown et al. Took a decade to finish.
- Data from both proton and deuteron targets.
- Cross section also depends on the axial correction

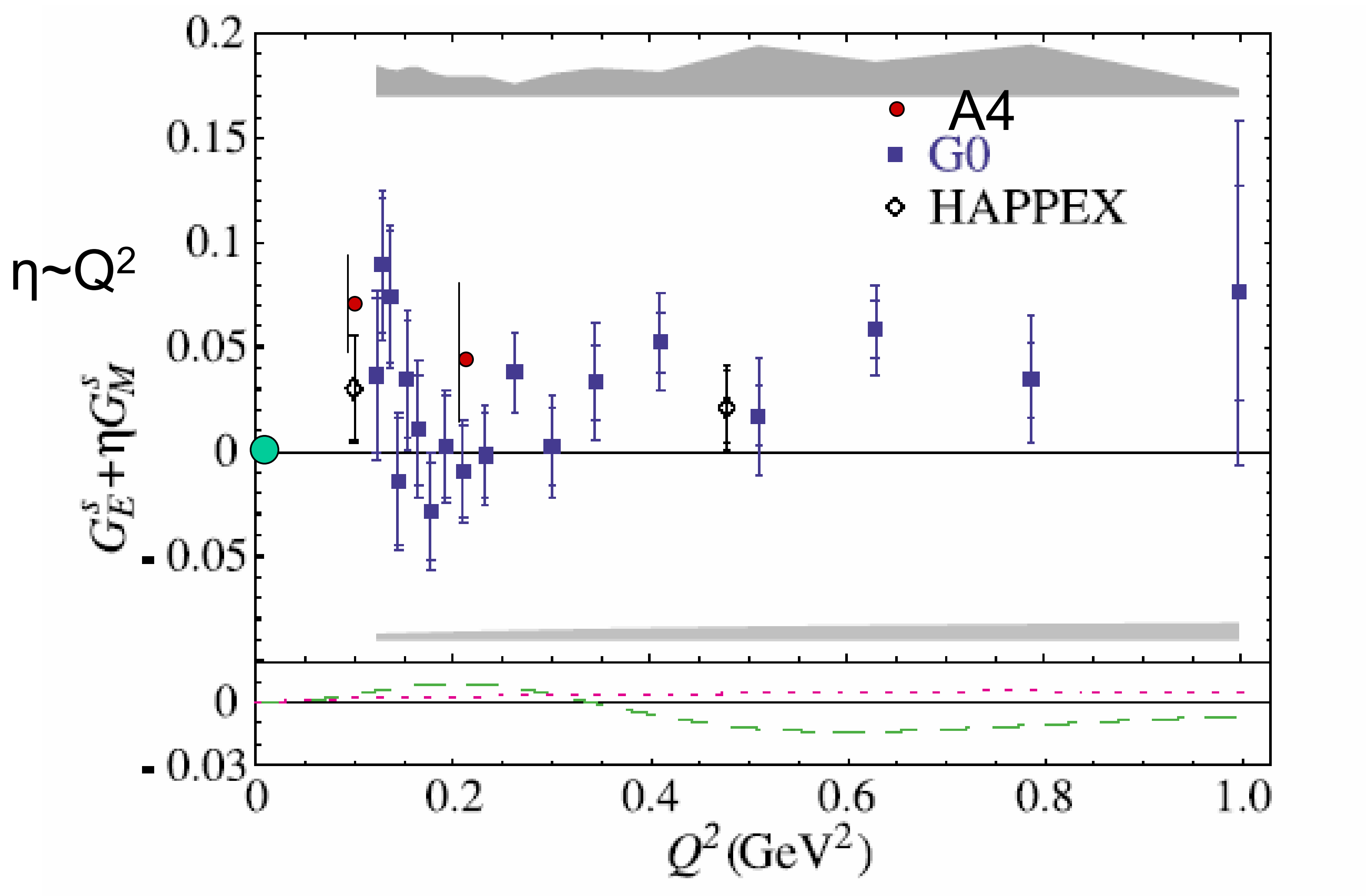


$$
\begin{aligned}
G_M^s &= 0.23 \pm 0.36 \pm 0.40 \\
G_A^{e\,(T=1)} &= -0.53 \pm 0.57 \pm 0.50\,,
\end{aligned}
$$

Review:

E. Beise et.al, Nucl-ex/0412054

Happex, PVA4, and G0 Results



GO conclusions

- Non-strange hypothesis is disfavored at 89% confidence.
- Initial rise from Q^2 = 0.0 to 0.05 GeV^2 is consistent with SAMPLE & HAPPEX & PVA4

$$G_M{}^s(Q^2 = 0.1\ \mathrm{GeV}^2) = 0.5$$

- G_E may be negative at Q^2 = 0.3 GeV^2 (PVA4)
- The combination is positive at large Q^2. (HAPPEX)

Magnetic moment as a function of mass

- When the mass of the strange quark is light, chiral-perturbation theory indicates that the magnetic moment contribution is negative.
- Consider K+Λ components. K+ is in a p-wave with one-unit of orbital angular momentum.
- s-bar orbital contribution to the magnetic moment from K+ is negative.
- The spin contribution from Λ is also negative.

Lattice QCD---An Indirect Approach

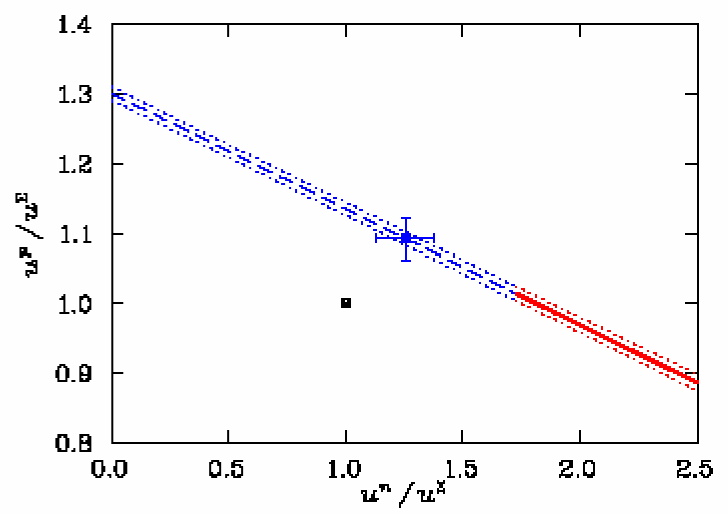
- D. B. Leinweber et. al. PRL94, 212001 (2005)

Find two expressions

$$G_M^s = \left(\frac{{}^{\ell}R_d^s}{1 - {}^{\ell}R_d^s}\right)\left[3.673 - \frac{u^p}{u^{\Sigma}}(3.618)\right],$$

$$G_M^s = \left(\frac{{}^{\ell}R_d^s}{1 - {}^{\ell}R_d^s}\right)\left[-1.033 - \frac{u^n}{u^{\Xi}}(-0.599)\right],$$

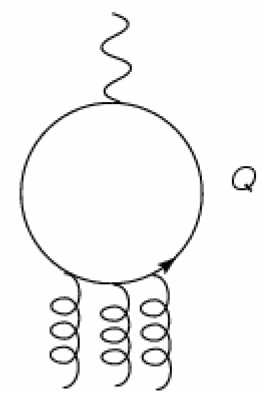
R is the ratio of strange to down quark sea contribution.



- They assume 0<R<1, and hence G_s must be negative!
- From a model, they find R= 0.139, hence μ_s = -0.046!
- However, R could be a number less than 0 or larger than 1. if R=−0.5, G_s = 0.2!

A heavy strage quark

- Consider a heavy quark contribution to the electromagnetic current



$$j_\mu^Q = \frac{g_s^3}{(4\pi)^2 45 m_Q^4}\partial^\alpha \left[14\mathrm{Tr}G^{\alpha\beta}\{G_{\mu\alpha}, G_{\beta\nu}\} + 5\mathrm{Tr}G_{\mu\nu}\{G^{\alpha\beta}, G_{\alpha\beta}\}\right]$$

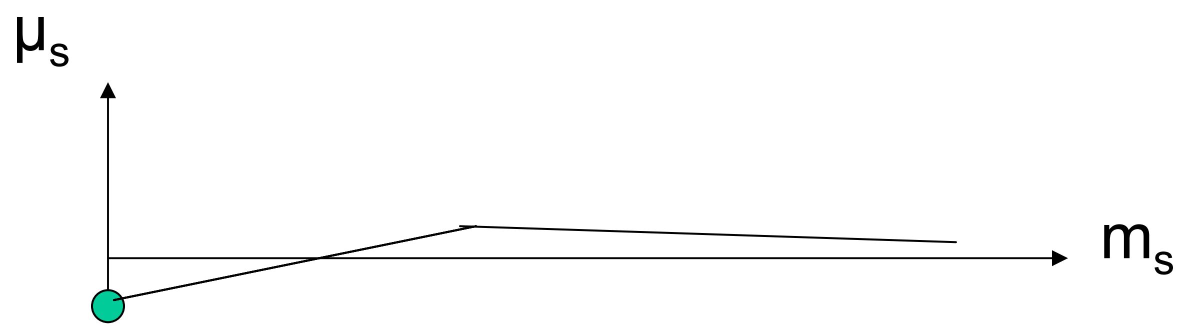
Contribution to the magnetic moment

- Contribution to the magnetic moment

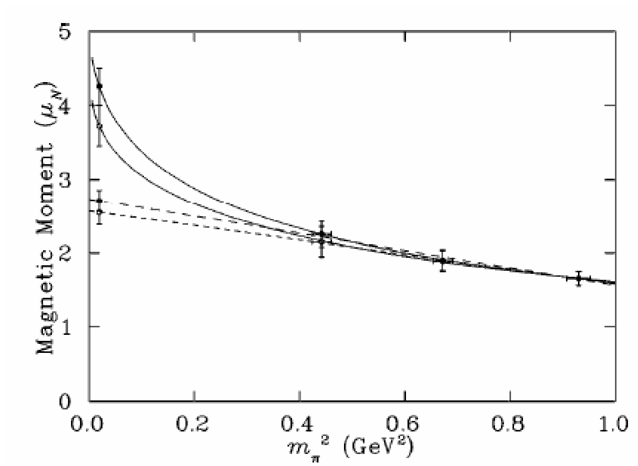
$$\mu_Q = \frac{g_s^3}{(4\pi)^2 45 m_Q^4} \sum d^{abc} \left\langle P \uparrow \left| \left[7(\vec{E}^a \cdot \vec{B}^b) E^{zc} - 2(\vec{E}^a \cdot \vec{E}^b - \vec{B}^a \cdot \vec{B}^b) B^{zc} \right] \right| P \uparrow \right\rangle$$

- In quark and bag model calculations, the heavy quark contribution is positive, similar to the argument leading to Δg is positive.
- Therefore the magnetic moment from strange quarks varies from negative to positive as mass runs from light to heavy compared to Λ.

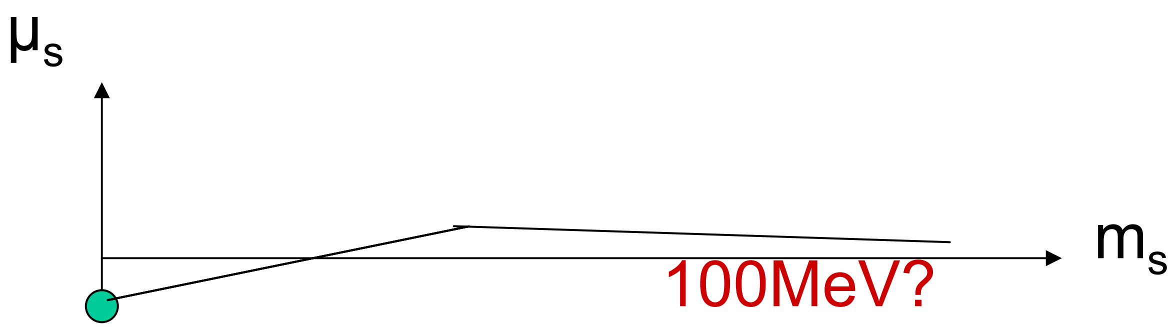
Magnetic moment as a function of mass



- Magnetic moment has a strong chiral dependence. The chiral behavior sets in at very small quark mass.



Magnetic moment as a function of mass



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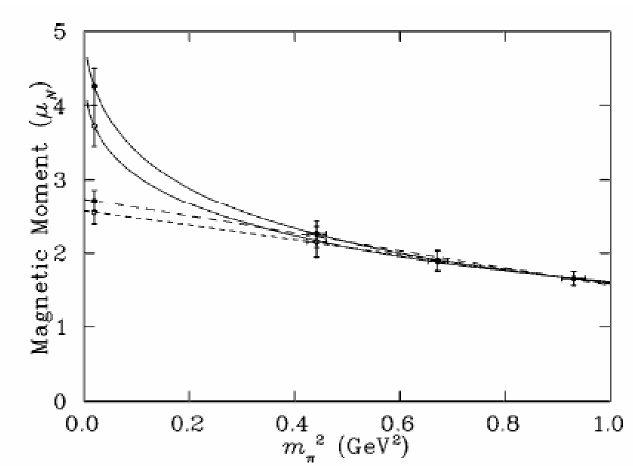

Conclusion

- The polarization of the proton leads to polarization of the color electric and magnetic field. There are many probes which allow us to establish a coherent picture
  - Δg
  - SSA
  - g2, higher-twist correction to g_1
  - Magnetic moment